Using the definition of the derivative, prove the derivative of $\cos x$. **SCORE: _____ / 15 PTS**

NOTE: You may use the values of the two special limits proved in class without reproving them, if you state the values of those limits.

$$\lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \lim_{h\to 0}\left((\cos x)\frac{\cosh - 1}{h} - (\sin x)\frac{\sinh}{h}\right)$$

$$= \lim_{h\to 0} \cos x \lim_{h\to 0} \frac{\cosh - 1}{h} - \lim_{h\to 0} \sin x \lim_{h\to 0} \frac{\sinh}{h}$$

$$= (\cos x)(0) - (\sin x)(1) = -\sin x$$

MULTIPLE CHOICE. Circle the correct answer. Show work to prove your answer is correct. **SCORE: _____ / 20 PTS**

Which of the following curves is orthogonal to the curve $y = \ln(\csc x + \cot x)$?

[a] $y = \sin x$ [b] $y = -\sin x$ [c] $y = \cos x$ [d] $y = -\cos x$ [e] $y = \sin x - \tan x$

$$\frac{dy}{dx} = \frac{1}{\csc x + \cot x}(-\csc x \cot x - \csc^2 x)$$

$$= \frac{-\csc x(\cancel{\cot x + \csc x})}{\cancel{\csc x + \cot x}}$$

OTHER $\frac{dy}{dx} = -\frac{1}{-\csc x} = \sin x$

If $f(x) = \dfrac{6x^2 - 9x}{\sqrt[3]{x}}$, find $\dfrac{d^3y}{dx^3}$.

SCORE: _____ / 15 PTS

$$f(x) = 6x^{\frac{5}{3}} - 9x^{\frac{2}{3}}$$

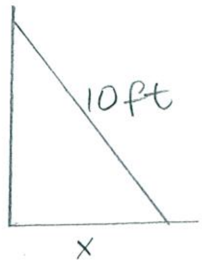
$$\frac{dy}{dx} = 10x^{\frac{2}{3}} - 6x^{-\frac{1}{3}}$$

$$\frac{d^2y}{dx^2} = \frac{20}{3}x^{-\frac{1}{3}} + 2x^{-\frac{4}{3}}$$

$$\frac{d^3y}{dx^3} = -\frac{20}{9}x^{-\frac{4}{3}} - \frac{8}{3}x^{-\frac{7}{3}}$$

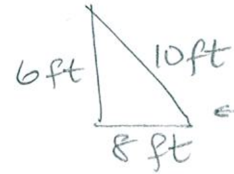
A 10 foot long ladder leans against a wall. If the bottom of the ladder is being pulled away from the wall at 2 ft per second, how quickly is the area between the ladder, the wall and the ground changing when the top of the ladder is 6 ft above the ground ? **SCORE: _____ / 25 PTS**

NOTE: Give the units of your final answer. State clearly whether the area is growing or shrinking.



$$\frac{dx}{dt} = 2 \text{ ft/sec}$$

WANT $\left.\dfrac{dA}{dt}\right|_{x = 8 \text{ ft}}$



$A = 24 \text{ ft}^2$

$$A = \frac{1}{2} x \sqrt{100 \text{ ft}^2 - x^2}$$

$$4A^2 = x^2 (100 \text{ ft}^2 - x^2)$$

$$= 100 x^2 \text{ ft}^2 - x^4$$

$$8A \frac{dA}{dt} = (200 x \text{ ft}^2 - 4x^3) \frac{dx}{dt}$$

$$\cancel{8} (24 \cancel{\text{ft}^2}) \frac{dA}{dt} = (200 (\cancel{8} \text{ ft}) \cancel{\text{ft}^2} - 4 (\cancel{8} \text{ ft})^{\cancel{3}}) (2 \text{ ft/sec})$$

$$\overset{3}{\cancel{24}} \frac{dA}{dt} = (200 \text{ ft} - 256 \text{ ft})(2 \text{ ft/sec}) = -\overset{-14}{\cancel{112}} \text{ ft}^2/\text{sec}$$

$$\frac{dA}{dt} = -\frac{14}{3} \frac{\text{ft}^2}{\text{sec}}$$

THE AREA IS SHRINKING

If $f(x) = (\tan x)^{\cos^{-1} x}$, find $f'(x)$. **SCORE: _____ / 25 PTS**

$$\ln f(x) = \cos^{-1} x \ln \tan x$$

$$\frac{1}{f(x)} f'(x) = -\frac{1}{\sqrt{1-x^2}} \ln \tan x + \cos^{-1} x \frac{1}{\tan x} \sec^2 x$$

$$f'(x) = (\tan x)^{\cos^{-1} x} \left(\frac{\cos^{-1} x \sec^2 x}{\tan x} - \frac{\ln \tan x}{\sqrt{1-x^2}} \right)$$

Find the **equation of the normal line** to the curve $\tan^{-1}(5x+y^2)=\dfrac{\pi}{2y}$ at $(1,-2)$. **SCORE: _____ / 25 PTS**

$$5x+y^2=\tan\frac{\pi}{2y}$$

$$5+2y\frac{dy}{dx}=\left(\sec^2\frac{\pi}{2y}\right)\left(-\frac{\pi}{2y^2}\frac{dy}{dx}\right)$$

$$5-4\left.\frac{dy}{dx}\right|_{(1,-2)}=\left(\sec^2\left(-\frac{\pi}{4}\right)\right)\left(-\frac{\pi}{8}\left.\frac{dy}{dx}\right|_{(1,-2)}\right)$$

$$5-4\left.\frac{dy}{dx}\right|_{(1,-2)}=2\cdot\frac{-\pi}{8}\left.\frac{dy}{dx}\right|_{(1,-2)}$$

$$5=\left(4-\frac{\pi}{4}\right)\left.\frac{dy}{dx}\right|_{(1,-2)}$$

$$\left.\frac{dy}{dx}\right|_{(1,-2)}=\frac{5}{4-\frac{\pi}{4}}$$

$$y+2=\frac{\frac{\pi}{4}-4}{5}(x-1)$$

$$y+2=\frac{\pi-16}{20}(x-1)$$

Let $f(x) = \dfrac{e^{2x-2}}{3^x - 1}$.

SCORE: _____ / 25 PTS

[a] If x changes from 1 to 1.2, find dy.

$$\frac{dy}{dx} = \frac{2e^{2x-2}(3^x-1) - e^{2x-2}(3^x \ln 3)}{(3^x-1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{2e^0(3^1-1) - e^0(3^1 \ln 3)}{(3^1-1)^2} = \frac{4-3\ln 3}{4}$$

$$dy = \frac{4-3\ln 3}{4}(1.2-1) = \frac{4-3\ln 3}{20}$$

[b] Approximate $f(1.2)$.

$$y(1) + dy = \frac{e^0}{3^1-1} + \frac{4-3\ln 3}{20}$$

$$= \frac{1}{2} + \frac{4-3\ln 3}{20}$$

$$= \frac{14-3\ln 3}{20}$$